2/7/19 - 1

TOSS COIN $P(H) = \frac{2}{3}$

WANT P(1ST HEAD HAPPENS ON 2nd TOSS)

⇒ SEE TAIL THEN HEAD

$P(TH) = \frac{1}{3} \cdot \frac{2}{3}$

$\frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9}$

EXPECTED VALUE

BERNOULLI $P(H) = p$ $P(T) = 1-p = q$

INDICATOR $H \to 1$ $T \to 0$

$P(1) = p$ $P(0) = 1-p$

$E[X] = p \cdot 1 + (1-p) \cdot 0 = p$

BINOMIAL $n = 2$ $p = \frac{1}{3}$

k = # HEADS $P(2) = 1/9$ $P(1) = \frac{4}{9}$

$E[X] = \frac{1}{9} \cdot 2 + \frac{4}{9} \cdot 1 + \frac{4}{9} \cdot 0$

$= \frac{2}{9} + \frac{4}{9} = \frac{6}{9} = \frac{2}{3}$

X: TOSS FAIR COIN $p = \frac{1}{2}$ $E[X] = \frac{1}{2}$

DEFINE $Y = 10X + 3$

$E[Y]$ $P_Y[3] = \frac{1}{2} = P_Y[13]$

$E[Y] = 8$

$E[Y] = 10E[X] + 3$

$\frac{1}{2}$

$E[aX + bY + c] = aE[X] + bE[Y] + c$

SAMPLING WITH REPLACEMENT

- 2 CARDS ARE INDEPENDENT

$P[1^{st} \text{ IS } H] = \frac{1}{4}$

$P[2^{nd} \text{ IS } H] = \frac{1}{4}$

$P[1 \text{ IS HEART}] = \frac{1}{4} \cdot \frac{3}{4} + \frac{3}{4} \cdot \frac{1}{4} = \frac{3}{8}$

VARIANCE

$$VAR[X] = E[(X - m_x)^2]$$

X = UNIFORM, $n = 4$ $P_X(0) = \frac{1}{4}$ $P(1)$ ...

$$E[X] = \frac{1}{4}\cdot 0 + \frac{1}{4}\cdot 1 + \frac{1}{4}\cdot 2 + \frac{1}{4}\cdot 3 = \frac{3}{2}$$

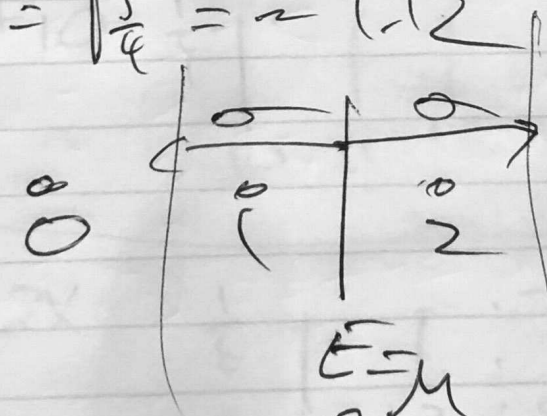
$$VAR[X] = \sum p(x_i)\left(x_i - \frac{3}{2}\right)^2$$

~~$= \frac{1}{4}\left(\frac{25}{4} + \frac{9}{4} + \right.$~~

$$\frac{1}{4}\left(\frac{9}{4} + \frac{1}{4} + \frac{1}{4} + \frac{9}{4}\right) = \frac{20}{16} = \frac{5}{4}$$

~~$STD = \sqrt{VAR} = \sqrt{20} = 4\frac{1}{2}$~~

$$STD = \sqrt{\frac{5}{4}} = \sim 1.12$$



$$VAR[X] = E[X^2] - (E[X])^2$$

$$= \frac{14}{4} - \frac{9}{4} = \frac{5}{4}$$

$$VAR[X] = E\left[(X - E[X])^2\right]$$

$$= E\left[X^2 - 2XE[X] + E[X]^2\right]$$

$$= E[X^2] - 2E[2XE[X]] + E[E[X]^2]$$

$$= E[X^2] - 2E[X]E[X] + (E[X])^2$$

$$= E[X^2] - (E[X])^2$$

$X : U[0,3]$ $n=4$ $\frac{1}{4}$ | | | | $\mu = \frac{3}{2}$

$VAR = \frac{5}{4}$ 0 1 2 3 $\sigma = \sqrt{\frac{5}{4}}$

$Y = X + 10$ $\frac{1}{4}$ | | | | $\mu = 11\frac{1}{2}$

10 11 12 13 $\sigma = \sqrt{\frac{5}{4}}$

$Z = 2X$ $\frac{1}{4}$ | | | | $\mu = 3$

0 2 4 6 $\sigma = \sqrt{5}$

$VAR = 5$

$E[Z^2] = \frac{1}{4}(0+4+16+36) = 14$

$E[Z] = 3$ & $VAR = 14 - 9 = 5$

BERNO.

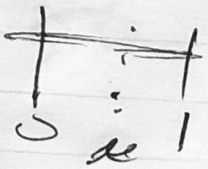
$P[0] = 1-p \quad p[1] = p$

$E[x] = p \cdot 1 + (1-p)\cdot 0 = p$

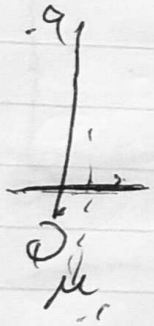
$E[x^2] = p \cdot 1 + (1-p)0 = p$

$VAR[x] = p - p^2$

IF $p = \frac{1}{2}$ $VAR = \frac{1}{4}$ $\sigma = \frac{1}{2}$



UNFAIR COIN $p = .1$ $\mu = .1$ $VAR = .09$ $\sigma = .3$

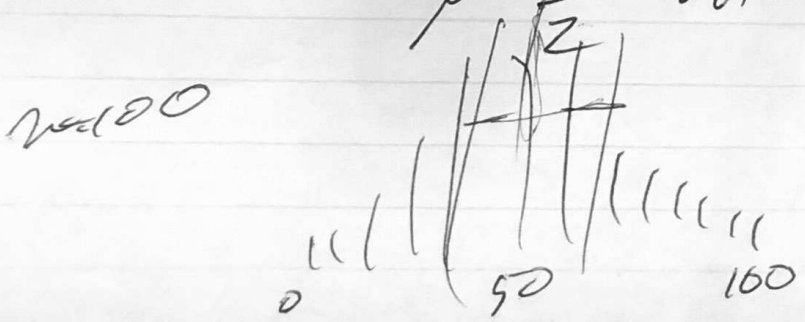


TOSS COIN n TIMES.

NOW $n = \pm$ TOSSES $\mu = np$

$VAR = npq$

FAIR: $\mu = \frac{n}{2}$ $VAR = \frac{n}{4}$ $\sigma = \frac{\sqrt{n}}{2}$

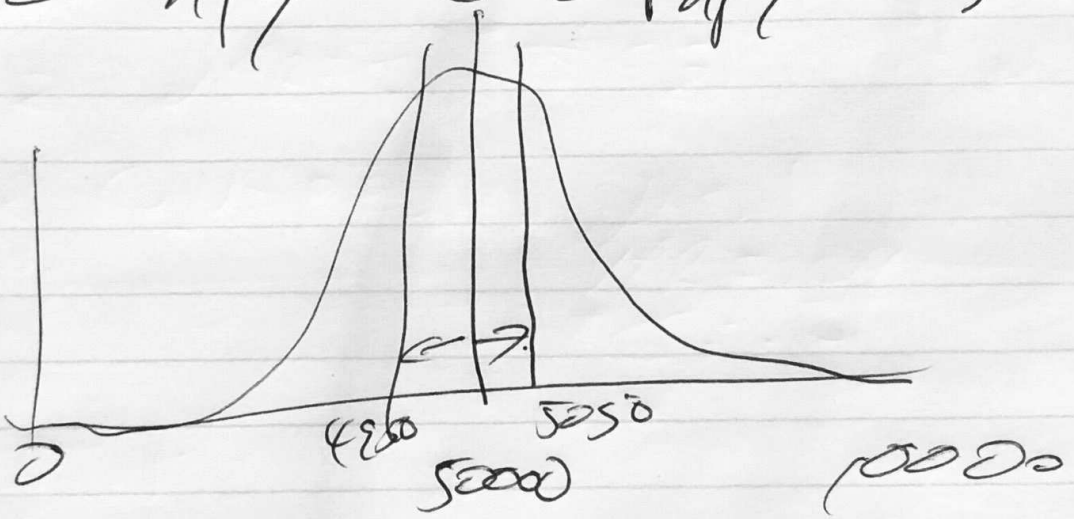


Toss fair coin 10000 times

$\mu = np = 10000\left(\frac{1}{2}\right) = 5000$

$\sigma = \sqrt{npq} = 100$

$VAR = npq \qquad \sigma = \sqrt{npq} = 50$



$n = 100 \qquad p = \frac{1}{4} \qquad q = \frac{3}{4}$

$\mu = np = 25 \qquad VAR = npq = 100 \cdot \frac{1}{4} \cdot \frac{3}{4}$

$= \frac{300}{16} = \frac{75}{4}$