2/11/19 - 1

GEOMETRIC KEEP TRYING UNTIL SUCCESS

X.

P EACH SUCCESS = p.

$P[1^{st}$ SUCC ON k^{th} TRY$] = q^{k-1}p$

$q = 1 - p$

$$E[X] = \sum_{k=1}^{\infty} k\,p(k) = \sum k q^{k-1} p = p \sum k q^{k-1}$$

$$\frac{1}{1-q} = \sum_{k=0}^{\infty} q^k$$

$(1-q)^{-1}$

$$+(1-q)^{-2} = \sum_{k=1}^{\infty} k q^{k-1}$$

$$E[X] = p\left(+\underbrace{(1-q)}_{p}^{-2}\right) = \frac{\cancel{p}}{p^2} = \frac{1}{p}$$

2 TYPES OF WIDGETS

$P[\text{GOOD TYPE I}] = \alpha$

$P[\text{BAD II}] = 1-\alpha = \bar{\alpha}$

BOTH: LIFETIMES ARE GEOMETRIC DIST

$P_I[k] = P[k \mid I] = (1-r)^{k-1} r$

$P[X=k \mid II] = (1-s)^{k-1} s \qquad s > r$

WANT COMBO PROB

$P[X=k] = P[X=k \mid I]\,P(I) + P[X=k \mid II]\,P[II]$

$= \alpha (1-r)^{k-1} r + (1-\alpha)(1-s)^{k-1} s$

$E[K] = \alpha E[K \mid I] + \bar{\alpha} E[K \mid II]$

$\cancel{r = \frac{1}{2}, \; r = \frac{1}{4}, \; s = \frac{1}{2}, \; \alpha = \frac{1}{2}}$

$r = \frac{1}{2} \qquad s = \frac{3}{4} \qquad \alpha = \frac{1}{2}$

$$P[X=k] = \frac{1}{2}\left(1-\frac{1}{2}\right)^{k-1}\frac{1}{2} + \frac{1}{2}\left(\frac{1}{4}\right)^{k-1}\frac{3}{4}$$

$$= \frac{1}{2^{k+1}} + \frac{3}{2}\frac{1}{4^k}$$

$$P[k=1] = \frac{1}{4} + \frac{3}{2\cdot 4} = \frac{5}{8}$$

$$P[k=2] = \frac{1}{8} + \frac{3}{32} = \frac{7}{32}$$

VARIANCE OF GEOM.

$$\cancel{E[X]}\ VAR[X] \triangleq E[(X-\mu)^2]$$

$$= E[X^2] - (E[X])^2$$

$$P(k) = q^{k-1}p \qquad E[X] = \frac{1}{p}$$

$$E[X^2] = \sum k^2 p(k) = \sum_{k=1}^{\infty} k^2 q^{k-1} p$$

$$\frac{1}{1-p} = \sum p^k$$

$$(1-p)^{-1} = \sum p^k$$

$$(1-p)^{-2} = \sum k p^{k-1}$$

$$2(1-p)^{-3} = \sum k(k-1) p^{k-2}$$

POISSON EXAMPLE

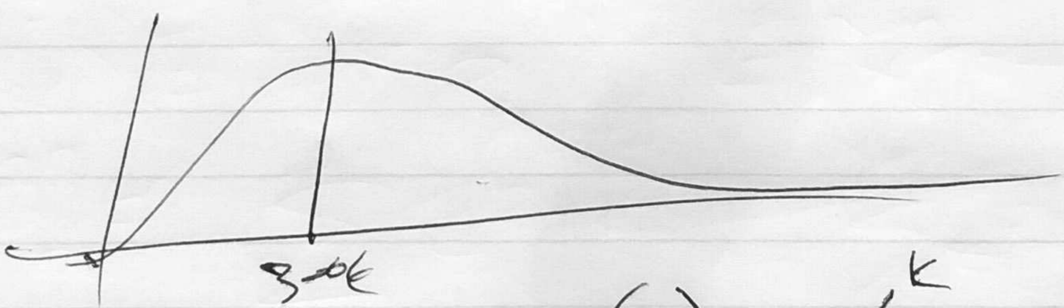
300M AMERICANS

P(BOB MAKES A PHONE CALL IN NEXT MIN)

$= \frac{1}{1000}$

EXPECTED # CALLS IS 300K

THE ACTUAL # IS POISSON $\alpha = 300K$



$$p(k) = \frac{\alpha^k}{k!} e^{-\alpha}$$

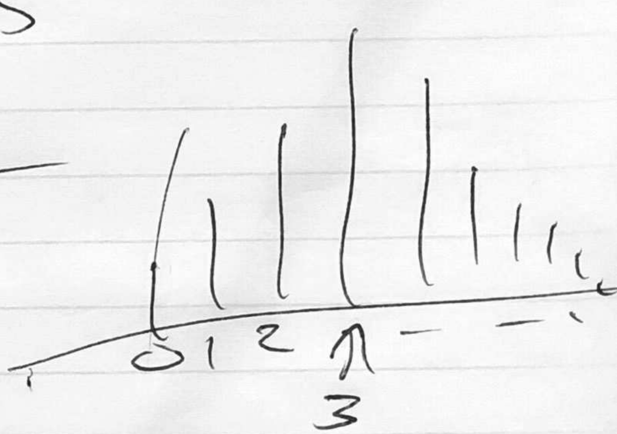
X: R.V. IS # PEOPLE IN CLASS PLAYING POKEMON GO NOW.

$$E[X] = 3 = \alpha$$

$$p(k) = \frac{3^k}{k!} e^{-3} \qquad e^{-3} = .05$$

$$p[0] = \frac{3^0}{0!} e^{-3} \approx .05$$

$$p[1] = \frac{3^1}{1!} e^{-3} = .15$$



FOR LARGE α, USE NORMAL APPROX.