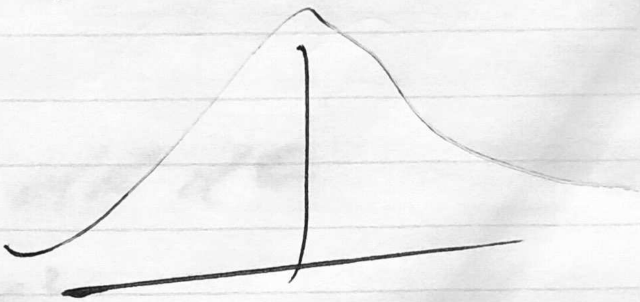
3/14/19 - 1

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$



PDF

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$A = \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}}\,dx$$

$$A^2 = \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}}\,dx \int_{-\infty}^{\infty} e^{-\frac{y^2}{2}}\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{x^2+y^2}{2}}\,dx\,dy$$

$x = R\cos\theta$

$y = R\sin\theta$

$dx\,dy = R\,dR\,d\theta$

$$A^2 = \int_0^\infty \int_0^{2\pi} R e^{-\frac{R^2}{2}}\,dR\,d\theta$$

$$= 2\pi \int_0^\infty R e^{-\frac{R^2}{2}}\,dR$$

$$= 2\pi \left. -e^{-\frac{R^2}{2}} \right|_0^\infty$$

$$= 2\pi$$

$$\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}}\,dx = \sqrt{2\pi}$$

$$\frac{1}{\sqrt{2\pi}} \int e^{-\frac{x^2}{2}}\,dx = 1$$

$\mu = 500 \quad \sigma = 100$

$P[400 \leq x \leq 600]$?

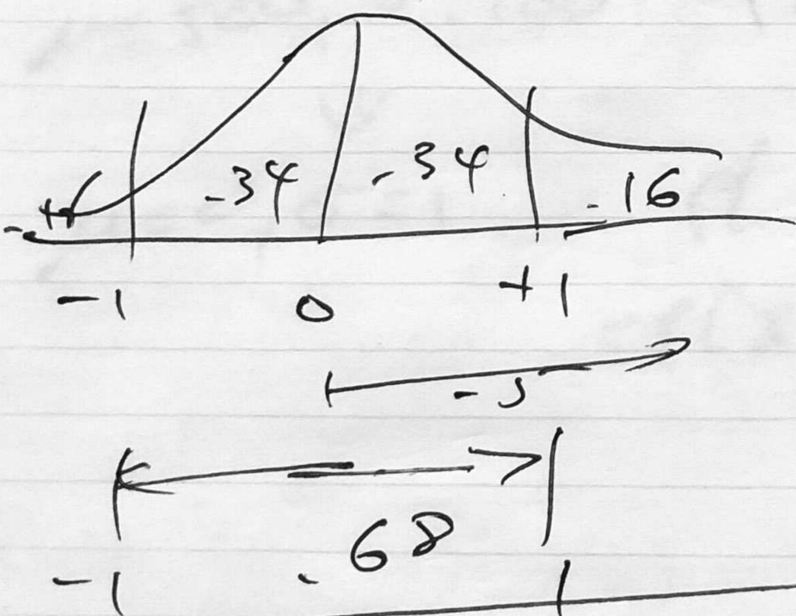
→ CHANGE THIS TO THE NORMAL FORM

$\mu = 0 \quad \sigma = 1$

$400 = \mu - \sigma$
$600 = \mu + \sigma$

$P[-1 \leq x \leq 1]$

IN TABLE $P[x \geq 1] = .16$



THE Q SAID $\mu = 500, \sigma = 100$

$400 = 500 - 100 = \mu - \sigma$

$\mu = 500, \sigma = 100$ WANT $P[500 \leq x \leq 700]$

$\parallel\parallel$

$\mu = 0, \sigma = 1$ $P[0 \leq x \leq 2]$

FROM TABLE, $PROB[x \geq 2] = .02$

$P[x \geq 0] = .5$

$P[0 \leq x \leq 2] = .5 - .02 = .48$

$\mu = 500, \sigma = 100$ $P[x \leq 300]$

$\Downarrow$

$\mu = 0, \sigma = 1$ $P[x \leq -2]$

$= P[x \geq 2] = .02$

EX 5.9 N BYTE MESSAGE

IT'S SPLIT INTO Q FULL BLOCKS (EACH IS M BYTES LONG) AND A R-BYTE FRAGMENT.

$N = qM + r$. $\quad q = \left\lfloor \frac{N}{M} \right\rfloor$ FLOOR.

~~INC~~ INT N, M, Q;

Q = N/M;

PMF OF N IS GEOMETRIC.

$p(n) = (1-p)\, p^{-n}$ $\quad$ p IS A PARAMETER.

CHECK $\displaystyle\sum_{n=0}^{\infty} p(n) = (1-p) \sum_{0}^{\infty} p^{-n} = \frac{1-p}{1-p} = 1$

$p(q, r) = (1-p)\, p^{-(qM+r)}$

$$P(n) = \sum_{q=0}^{\infty} p(q,n)$$

$$= (1-p)\sum_{q=0}^{\infty} p^{-(qM+n)}$$

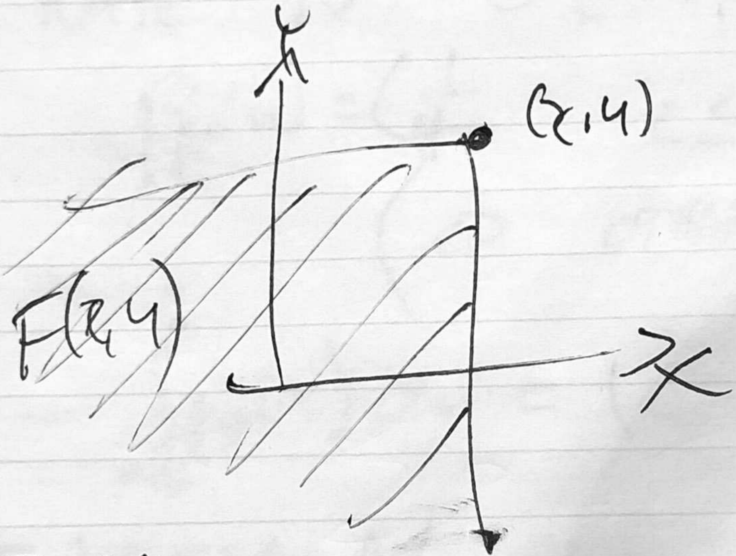
$$=(1-p)\,p^{-n}\sum_{q=0}^{\infty} p^{-qM}$$

$$\cancel{\frac{1}{1-qM}} \quad \frac{1}{1-p^{M}}$$

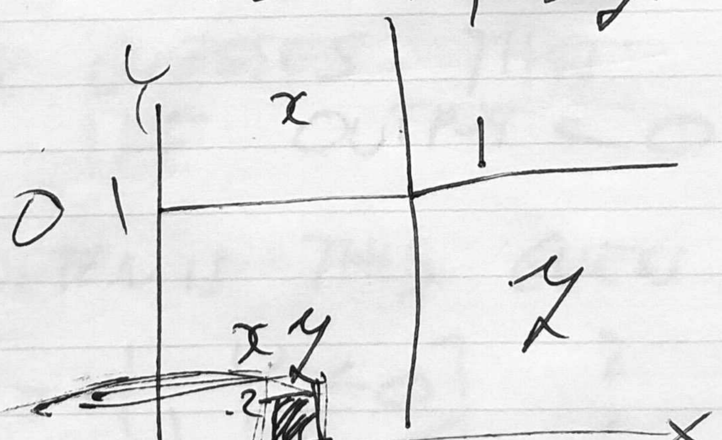
$$P(n) = \cancel{\frac{1-p}{1-qM}}\left(\frac{1-p}{1-p^{M}}\right)p^{-n}$$

CHECK $\displaystyle\sum_{n=0}^{M-1} P(n) = \frac{1-p}{1-p^{M}} \; \cdots \; = 1$

$$F_{X,Y}(x,y) = P[X \le x, Y \le y]$$



$$F(x,y) = xy \qquad 0 \le x \le 1, \; 0 \le y \le 1$$



$$P[0.5 \le x \le .6, \; 0 \le y \le .2]$$

$$= F(.6, .2) - F(.5, .2) = .12 - .10$$

$$= .02$$

EX 5.14 INPUT: X : -1 OR 1 50/50

NOISE N : $U[-2,2]$

$$P_N(x) = \begin{cases} \frac{1}{4} & -2 \le x \le 2 \\ 0 & \text{OTHERWISE} \end{cases}$$

$$\int_{-2}^{2} P_N(x)\,dx = 1$$

OUTPUT $Y = X + N$

RECEIVER GUESSES THAT INPUT WAS -1 IFF OUTPUT < 0

HOW OFTEN IS THIS GUESS WRONG?

$P[X = +1, Y \le 0]$?

$$= P[\underset{\substack{X+N \\ 1+N \\ N \le -1}}{Y \le 0} \mid X = 1]\, \underset{\frac{1}{2}}{P[X = 1]}$$

$$P[N \le -1]\, \underset{1/2}{P[X = 1]}$$

$N: -2 \longrightarrow 2$

$P[N \leq -1] = 1/4$

$P[X = +1, Y \leq 0] = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$