3/18/18 - 1

$Z = X + Y$

$f_{X,Y}(x,y)$ PDF

$E[Z] = \int z f(z)\, dz$

$\iint (x'+y') f(x',y')\, dx' dy'$

$= \iint \boxed{x' f(x',y')\, dx' dy'}$

$+ \iint y' f(x',y')\, dx' dy'$

$= E[X] + E[Y]$

WHETHER OR NOT X, Y INDEPENDENT

$X : U[0,1]$ CORRELATED

$E[X] = 1/2$

$Y = X$ $E[Y] = 1/2$

$Z = X + Y = 2X$ $Z : U[0,2]$

$E[Z] = 1$

$Y = 1 - X$ ANTICORRELATED.

$Z = X + Y = 1$

$E[X] = 1/2$ $E[Y] = 1/2$

$E[Z] = 1$

$X : U[0,1]$ $Y = X$

$E[X] = 1/2 = E[Y]$

$E[XY] = E[X^2] = \int x^2 f(x)\,dx = \frac{x^3}{3}\Big|_0^1$

$= 1/3$

$E[\cancel{E[A]}$

$$= E[(X - E[X])(Y - E[Y])]$$

$$= E[XY - XE[Y] - E[X]Y + E[X]E[Y]]$$

$$= E[XY] - E[X]E[Y]$$

$$= \frac{1}{3} - \frac{1}{4} = \frac{1}{12} = COV[X, Y]$$

ANTICOR $Y = -X$

$$E[X] = E[Y] = \frac{1}{2}$$

$$E[XY] = E[X(1-X)] = E[X] - E[X^2]$$

$$\frac{1}{2} \qquad \frac{1}{3}$$

$$COV[X, Y] = \frac{1}{6} - \frac{1}{4} = -\frac{1}{12}$$

$= 1/6$

X, Y INDEP

$$E[XY] = E[X]E[Y] = \frac{1}{4}$$

$$COV = 0$$

$X : U[0,1] \quad E[X] = 1/2$

$VAR = E[X^2] - E[X]^2 = E[X - E[X]]^2$

$\int_0^1 x^2 f(x)\,dx = \int x^2\,dx = \frac{x^3}{3} = \frac{1}{3}$

$VAR[X] = 1/3 \quad \sigma = \sqrt{\frac{1}{3}}$

$Y_i = X \quad COV[X, Y_i] = 1/12$

$\rho_{XY_i} = \rho_{XY_i} = \frac{COV[X, Y_i]}{\sigma_X \sigma_{Y_i}} = \frac{\frac{1}{12}}{\sqrt{\frac{1}{3}}\sqrt{\frac{1}{3}}} = \frac{1}{4}$

$VAR = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

$\rho_{XY_i} = \frac{COV}{\sigma_X \sigma_Y} = \frac{\frac{1}{12}}{\sqrt{\frac{1}{12}}\sqrt{\frac{1}{12}}} = 1$

EX 5.29 p261

5

$P_X(x=5) = \frac{1}{6}$

$P_X(x=5 \mid y=1) = \frac{P(x=5, y=1)}{P(y=1)} = \frac{1/42}{1/6}$

$= \frac{1}{7}$

$$\boxed{P(A|B) = \frac{P(A \& B)}{P(B)}}$$

$$P(n) = \sum_{q=0}^{\infty} p(q,n)$$

$$= (1-p)\sum_{q=0}^{\infty} p^{-(qM+n)}$$

$$= (1-p)\, p^{-n} \sum_{q=0}^{\infty} p^{-qM}$$

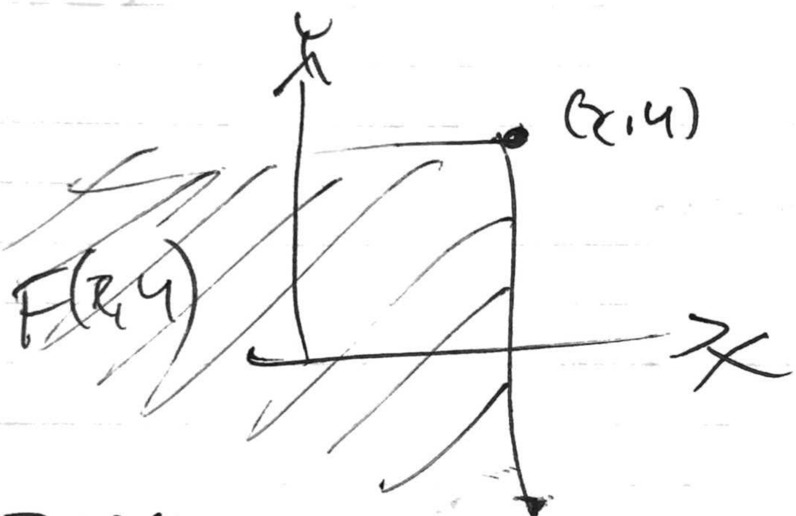
$$\cancel{\frac{1}{1-M}} \quad \frac{1}{1-p^M}$$

$$p(n) = \cancel{\frac{1-p}{1-q^M}} \left(\frac{1-p}{1-p^M}\right) p^{-n}$$

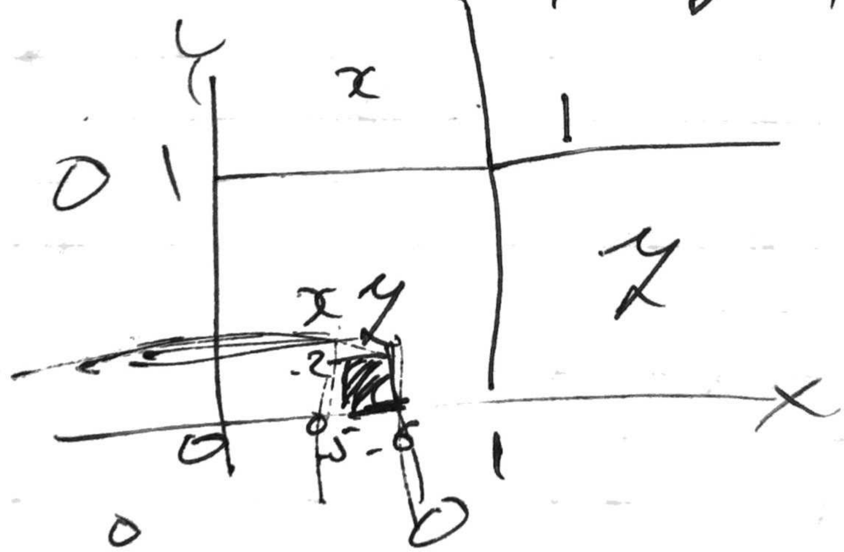
CHECK

$$\sum_{n=0}^{M-1} p(n) = \frac{1-p}{1-p^M} \quad \ldots \quad = 1$$

$$F_{X,Y}(x,y) = P[X \le x,\ Y \le y]$$



$$F(x,y) = xy \quad 0 \le x \le 1,\ 0 \le y \le 1$$



$$P[0.5 \le x \le .6,\ 0 \le y \le .2]$$
$$= F(.6,.2) - F(.5,.2) = .12 - .10$$
$$= .02$$

EX 5.14 50/50

INPUT: X : -1 OR 1

NOISE N : U[-2, 2]

$$P_N(x) = \begin{cases} \frac{1}{4} & -2 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\int_{-2}^{2} p_N(x)\, dx = 1$$

OUTPUT $Y = X + N$

RECEIVER GUESSES THAT INPUT WAS -1 IFF OUTPUT < 0.

HOW OFTEN IS THIS GUESS WRONG?

$P[X = +1, Y \le 0]$?

$$= P[\underset{\substack{X+N \\ 1+N \\ N \le -1}}{Y \le 0} \mid X = 1]\ \underset{\frac{1}{2}}{P[X=1]}$$

$$P[N \le -1]\ \underset{1/2}{P[X=1]}$$

$N: -2 \longrightarrow 2$

$P[N \leq -1] = 1/4$

$P[X = +1, Y \leq 0] = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$